MODULE 8
INTRODUCTION TO RISK AND RETURN, AND THE OPPORTUNITY COST OF CAPITAL

Revised by Gene Lai
Risk and Return

Risk and Return are related.

How?

This module will focus on risk and return and their relationship to the opportunity cost of capital.
Outline

- Rates of Return
- A Century of Capital Market History
- Measuring Risk
- Risk & Diversification
- Thinking About Risk
Equity Rates of Return: A Review

Percentage Return = \( \frac{\text{Capital Gain} + \text{Dividend}}{\text{Initial Share Price}} \)

Dividend Yield = \( \frac{\text{Dividend}}{\text{Initial Share Price}} \)

Capital Gain Yield = \( \frac{\text{Capital Gain}}{\text{Initial Share Price}} \)
Rates of Return: Example

Example: You purchase shares of GE stock at $15.13 on December 31, 2009. You sell them exactly one year later for $18.29. During this time GE paid $.46 in dividends per share. Ignoring transaction costs, what is your rate of return, dividend yield and capital gain yield?

\[
\text{Percentage Return} = \frac{\text{Selling Price} - \text{Purchasing Price} + \text{Dividends}}{\text{Purchasing Price}} = \frac{18.29 - 15.13 + 0.46}{15.13} = 23.93\%
\]

\[
\text{Dividend Yield} = \frac{\text{Dividends}}{\text{Purchasing Price}} = \frac{0.46}{15.13} = 3.04\%
\]

\[
\text{Capital Gain Yield} = \frac{\text{Selling Price} - \text{Purchasing Price}}{\text{Purchasing Price}} = \frac{18.29 - 15.13}{15.13} = 20.89\%
\]
Real Rates of Return

Recall the relationship between real rates and nominal rates:

$$1 + \text{real rate of return} = \frac{1 + \text{nominal rate of return}}{1 + \text{inflation rate}}$$

Example: Suppose inflation from December 2009 to December 2010 was 1.5%. What was GE stock’s real rate of return, if its nominal rate of return was 23.93%?

$$1 + \text{real return} = \frac{1.2393}{1.015} = 1.2210$$

$$\therefore \text{real return} = 22.10\%$$
Capital Market History: Market Indexes

- **Market Index** - *Measure of the investment performance of the overall market.*
  
- Dow Jones Industrial Average (The Dow)
  - Value of a portfolio holding one share in each of 30 large industrial firms.

- Standard & Poor’s Composite Index (S&P 500)
  - Value of a portfolio holding shares in 500 firms. Holdings are proportional to the number of shares in the issues.
Total Returns for Different Asset Classes

The Value of an Investment of $1 in 1900
Annual Total Returns, 1926-1998

<table>
<thead>
<tr>
<th>Category</th>
<th>Average Return</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small-company stocks</td>
<td>17.4%</td>
<td>33.8%</td>
<td></td>
</tr>
<tr>
<td>Large-company stocks</td>
<td>13.2</td>
<td>20.3</td>
<td></td>
</tr>
<tr>
<td>Long-term corporate bonds</td>
<td>6.1</td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td>Long-term government</td>
<td>5.7</td>
<td>9.2</td>
<td></td>
</tr>
<tr>
<td>Intermediate-term government</td>
<td>5.5</td>
<td>5.7</td>
<td></td>
</tr>
<tr>
<td>U.S. Treasury bills</td>
<td>3.8</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>3.2</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>
The Difference in Total Returns?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Average Annual Rate of Return</th>
<th>Average Premium (Extra Return versus Treasury Bills)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury bills</td>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>Treasury bonds</td>
<td>5.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Common stocks</td>
<td>11.4</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Risk Premium: Expected return in excess of risk-free return as compensation for risk.

Maturity Premium: Extra average return from investing in long- versus short-term Treasury securities.
Risk Premium: Example

Expected Market Return = Interest Rate on Treasury Bills + Normal Risk Premium

1981: 21.4% = 14% + 7.4%

2008: 9.6% = 2.2% + 7.4%

2012: 7.47% = 0.07% + 7.4%
Returns and Risk

We next show how to measure expected return and risk.
Measuring Expected Rate of Return (for a Single Stock)

\[ r = \text{or } E(r) = \text{expected rate of return.} \]

\[ E(r) = \sum_{i=1}^{n} r_i P_i. \]
Measuring Risk (for a Single Stock)

What is risk? Uncertainty

How can it be measured?

**Variance:** Average value of squared deviations from mean. A measure of volatility.

**Standard Deviation:** Square root of variance. Also a measure of volatility.
Standard Deviation of a Single Stock

\[ \sigma = \text{Standard deviation.} \]

\[ \sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2} \]

\[ \sigma = \sqrt{\sum_{i=1}^{n} (r_i - E(r))^2 P_i}. \]
There are five possible states of economy next year. The probability associated each state is presented below. In addition, the returns associated with each state is also presented below.

<table>
<thead>
<tr>
<th>Economy</th>
<th>Prob.</th>
<th>T-Bill</th>
<th>HT</th>
<th>Coll</th>
<th>USR</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.1</td>
<td>8.0%</td>
<td>-22.0%</td>
<td>28.0%</td>
<td>10.0%</td>
<td>-13.0%</td>
</tr>
<tr>
<td>Below avg.</td>
<td>0.2</td>
<td>8.0</td>
<td>-2.0</td>
<td>14.7</td>
<td>-10.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Average</td>
<td>0.4</td>
<td>8.0</td>
<td>20.0</td>
<td>0.0</td>
<td>7.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Above avg.</td>
<td>0.2</td>
<td>8.0</td>
<td>35.0</td>
<td>-10.0</td>
<td>45.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Boom</td>
<td>0.1</td>
<td>8.0</td>
<td>50.0</td>
<td>-20.0</td>
<td>30.0</td>
<td>43.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1.0</strong></td>
<td><strong>8.0</strong></td>
<td><strong>50.0</strong></td>
<td><strong>-20.0</strong></td>
<td><strong>30.0</strong></td>
<td><strong>43.0</strong></td>
</tr>
</tbody>
</table>
Do the returns of HT and Coll. move with or counter to the economy?

- HT: Moves with the economy, and has a positive correlation. This is typical.
  ➔ HT is for high tech

- Coll: Is countercyclical of the economy, and has a negative correlation. This is unusual.
  ➔ Coll is for collection agency
Calculate the Expected Rate of Return for HT

\[ r = \text{Expected rate of return.} \]

\[
E(r) = \sum_{i=1}^{n} r_i P_i.
\]

\[
E(r) = (-22\%)0.1 + (-2\%)0.20 + (20\%)0.40 + (35\%)0.20 + (50\%)0.1 = 17.4\%.
\]
<table>
<thead>
<tr>
<th></th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>HT</td>
<td>17.4%</td>
</tr>
<tr>
<td>Market</td>
<td>15.0</td>
</tr>
<tr>
<td>USR</td>
<td>13.8</td>
</tr>
<tr>
<td>T-bill</td>
<td>8.0</td>
</tr>
<tr>
<td>Coll.</td>
<td>1.7</td>
</tr>
</tbody>
</table>

HT appears to be the best, but is it really?
What’s the standard deviation of returns for each alternative?

\[ \sigma = \text{Standard deviation.} \]

\[ \sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2} \]

\[ \sigma = \sqrt{\sum_{i=1}^{n} (r_i - E(r))^2 P_i}. \]
Standard deviation for T-Bill

\[ \sigma = \sqrt{\sum_{i=1}^{n} (r_i - E(r))^2 P_i}. \]

\[
\sigma_{T\text{-bills}} = \left[ (8.0 - 8.0)^2 0.1 + (8.0 - 8.0)^2 0.2 \\
+ (8.0 - 8.0)^2 0.4 + (8.0 - 8.0)^2 0.2 \\
+ (8.0 - 8.0)^2 0.1 \right]^{1/2}
\]

\[ \sigma_{T\text{-bills}} = 0.0\%. \]
Standard deviation for HT

\[ \sigma_{HT} = \left[ (-.22 - .174)^2 0.1 + (-.02 - .174)^2 0.2 \\
+ (.2 - .174)^2 0.4 + (.35 - .174)^2 0.2 \\
+ (.50 - .174)^2 0.1 \right]^{1/2} \]

\[ \sigma_{HT} = 20.0\%. \]
# Expected Returns vs. Risk

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected return</th>
<th>Risk, $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HT</td>
<td>17.4%</td>
<td>20.0%</td>
</tr>
<tr>
<td>Market</td>
<td>15.0</td>
<td>15.3</td>
</tr>
<tr>
<td>USR</td>
<td>13.8*</td>
<td>18.8*</td>
</tr>
<tr>
<td>T-bills</td>
<td>8.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Coll.</td>
<td>1.7*</td>
<td>13.4*</td>
</tr>
</tbody>
</table>

*Seems misplaced.*
Assume a two-stock portfolio with $50,000 in HT and $50,000 in Collections. Calculate \( E(r_p) \) and \( \sigma_p \).
Portfolio Expected Return, $r_p$

$E(r_p)$ is a weighted average:

$$E(r_p) = \sum_{i=1}^{n} w_i r_i.$$ 

$E(r_p) = 0.5(17.4\%) + 0.5(1.7\%) = 9.6\%.$

Note: This is one method to calculate portfolio return.
Portfolio Return for 2 Assets

Portfolio rate of return = \left( \text{fraction of portfolio in first asset} \right) \times \left( \text{rate of return on first asset} \right) + \left( \text{fraction of portfolio in second asset} \right) \times \left( \text{rate of return on second asset} \right)

Note that we can calculate portfolio rate of return for each scenario first, please see next slide.
Alternative Method to Calculate Mean

What is Port. Return if the economy is booming? 15% = (50 + (-20))/2
Alternative Method to Calculate Mean

\[ r_p = (3.0\%)0.10 + (6.4\%)0.20 + (10.0\%)0.40 + (12.5\%)0.20 + (15.0\%)0.10 = 9.6\% . \]

Note: we can treat port. as a single security when calculating and variance.
Alternative Method to Calculate Standard Deviation

\[
\sigma_p = \left[ \begin{array}{c}
(3.0 - 9.6)^2 \cdot 0.10 \\
+ (6.4 - 9.6)^2 \cdot 0.20 \\
+ (10.0 - 9.6)^2 \cdot 0.40 \\
+ (12.5 - 9.6)^2 \cdot 0.20 \\
+ (15.0 - 9.6)^2 \cdot 0.10
\end{array} \right]^{1/2} = 3.3\%.
\]
Some Comments

- $\sigma_p = 3.3\%$ is much lower than that of either stock (20\% and 13.4\%).

- $\sigma_p = 3.3\%$ is lower than the average of HT and Coll’s standard deviation $= (.5)(20\%) + (.5)(13.4\%) = 16.7\%$.

- The Portfolio provides average r but lower risk.

- Reason: negative correlation for this specific case.
Returns Distribution for Two Perfectly Negatively Correlated Stocks (-1.0) and for Portfolio WM

$\sigma_{WM} = \text{correlation coefficient between W and M}$
Returns Distributions for Two Perfectly Positively Correlated Stocks (+1.0) and for Portfolio MM’
Risk and Diversification

Diversification

Strategy designed to reduce risk by spreading a portfolio across many investments.

Unique Risk:

Risk factors affecting only that firm. Also called “diversifiable risk.”

Market Risk:

Economy-wide sources of risk that affect the overall stock market. Also called “systematic risk.”
Correlation Coefficient and Diversification

- Maximum diversification: correlation of coefficient = -1
- No diversification: correlation of coefficient = +1
- Correlation of coefficient < 1, there is still diversification.
Risk and Diversification

- Specific risk decreases as the number of securities increases.
- Market risk remains constant regardless of the number of securities.
Thinking About Risk

- **Message 1**
  - Some Risks Look Big and Dangerous but Really Are Diversifiable

- **Message 2**
  - Market Risks Are Macro Risks

- **Message 3**
  - Risk Can Be Measured