Management and Operations 340: Inventory Management and the EOQ Model

[Chuck Munson]: Hi, this is Chuck Munson. Today we're going to talk about the wonderful world of inventory management. Inventory may not sound like the most exciting subject in the world to you, but it's extremely important. Companies can lose and waste millions of dollars if they make poor inventory choices, and if you've ever thrown a party and run out of liquid refreshments for your guests, you know how problematic that might become. Same thing for companies: if they don't have the right amount at the right place at the right time, they may lose business, be penalized significantly, and if they have too much it's tremendously expensive, so inventory theory is all about finding the right balance as best we can, and chapter 12 in the book covers it in quite a bit of detail.

In this clip I want to just talk about some of the basics of inventory theory, some of the pros and cons and then look at what is called the economic order quantity model, which is the core base model that really all inventory models beyond that start from, and there have been- there are literally thousands of research papers on inventory theory. I've written some myself, but this EOQ model has been around over a hundred years, and it really captures the most important features.

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Inventory

Inventory serves as a buffer between supply and demand processes that do not fit nearly together, to mitigate the costly disruptions that would otherwise occur.

Example of a nearly "perfect" supply process: supply of cold water to homes

Specific Purposes

- Meet Anticipated demand
- Decouple production & distribution
  - Permits constant production quantities
- Permit smooth operations through work-in process (WIP) inventory, i.e. decouples operations
- Take advantage of quantity discounts
- Provide hedge against inflation
- Exploit economies of scale in supply
- Protect against shortages

[Chuck Munson]: Okay, so what is inventory? Well, it serves as a buffer between supply and demand processes that do not fit neatly together to mitigate the costly disruptions that would otherwise occur. In other words, you need to have enough around just in case you need it, so at home we usually keep extra milk on hand, extra meat, cheese, cereal, so if we get hungry or somebody comes by we have something to eat or drink. At the same time, we don't have a closet full of cereal because that would take a lot of space that we need for other things, right?

A nearly perfect supply process is a supple of cold water at homes. If you think about it, it's really quite amazing and it's been around for I guess hundreds of years, but if you turn on the

faucet you get water. You get as much water as you need, but it's stored in some sense underground, but it's not in your house. It's not like the old days where you had to go to the lake or to the well and carry, you know, gallons on your back and back-and-forth. It's just a nearly perfect process where you don't see the inventory but you get the product as soon as you need it. Hot water is a little different because you will run out if you keep it on and your hot water heater gets empty. But the world is not Star Trek yet; we can't replicate things, so we have to have real product on hand in order to do that. I guess 3-D printing is starting to look like Star Trek, but we're still a long way from being able to produce our Earl Gray Tea instantly from nothing with a replicator. So we use inventory instead.

Number of specific purposes for it: one is to meet anticipated demands, so an example of this is a buildup for Christmas. I know that Walmart will start stocking for Christmas in September sometimes, so it's kind of strange to go into a Walmart warehouse and see Santa Claus stuff, but they cannot get it all delivered in, you know, late November, December quick enough, efficiently enough, to get it to the stores, so they actually start putting it in the warehouse well ahead of time.

Another reason is to decouple production and distribution, so we're talking here about our suppliers and customers. *[Chuck Munson writes examples on screen]* So if you have raw materials inventory, you don't have to rely on your supplier to get you the goods there today. You can start working on something, and similarly, if you have finished goods inventory, you can make it, put it in your warehouse, and keep making it. It's not blocking anything, okay? That's kind of company to company within the operation itself. If you have work in process inventory in between stations, that means you always have something to work on because there's a bucket of material, and there's somewhere to put what you finished so your machine is not blocked, and that allows each machine to work at its full potential all the time, and so we have no idle machines.

Another big reason to have inventory is quantity discounts. As you know you may be offered a very big discount on price if you buy a lot so you end up buying more than you would otherwise to get that good price- it becomes inventory. Same idea as when you go to Costco and buy much more Spaghettios than you ever though you would use.

It can provide a hedge against inflation. In other words, if you buy- if you know inflation is coming or if your supplier tells you their prices are going to rise next month, you can buy before the price goes up. When I moved out to Pullman, Washington from Saint Louis, Missouri, one of the impacts was gasoline is more expensive out here, so I could have taken a bunch of gas cans and filled them up with gas and drove out. That would have been a way to have inventory to hedge against inflation. I didn't do it for the obvious environmental and safety concerns, but that would have been a way and a good use of inventory.

Another thing is exploiting the economies of scale in supply, and this is what we mean by cycle inventory, and this is really what we're focusing on very much in chapter 12: how much to order every time you order, and the idea is because you get some economies of scale by making more than one thing at a time or ordering more than one thing at a time- just tells you how much to order or make.

And finally inventory helps protect against shortages, so we call this safety stock. You have extra inventory on hand just in case something happens unexpectedly, and this could include weather, problems that keep the trucks from coming, other supply delays, quality problems- you get a shipment with defective goods. Or on the demand side, high-demand, and that's the idea of putting extra beer in your refrigerator before your Super Bowl party just in case people are very happy that their team won and they drink more or they're very sad that their team lost so they drink to fulfill their sorrows or, you know, people use whatever excuse they come up with.

Okay, what are some disadvantages of inventory? Well the ones we focus on the most are that it costs money. It costs money every time your order and it costs money for everything you hold in inventory. It's difficult to control: the more you have the more you have to store, maintain, keep records, etc.

Also the more you handle inventory, that's all non-value added activity. The customer could care less how many times you ship it from one warehouse to the other, how many forklifts it went on, how long you had it inventory- none of that adds value, so every bit of labor you use on inventory is costing you money but you can't charge more for that, okay?

It also reduces cash-availability, and this is particular true for small firms that don't have liquid financing options. If I own a little store of clothing and I have 200,000 dollars of inventory on my shelves and on my racks, that's 200,000 dollars I can't use to pay my bills and pay my workers. That's real money. Once I sell it I can, but if it's sitting in inventory, that's real money that I don't have to do something else.

The product might become obsolete. So having too much of it means, you know, it became out of style or something, you know, a new generation of smartphone- once the new generation comes out the old generation is worth much, much less on the market, so you may have to liquidate these things. And the other issue is you're less likely to respond to the market with new products if you have a whole bunch of old ones in your inventory, so that's a real problem with having too much around.

And it has production problems and your book shows an example of- this is a famous analogy that the Japanese provide us, and the idea there is that the rocks are bad things and the water is inventory, and inventory acts as kind of a double-edged sword: the more of it you have, the better able you are in the short-run to get over these problems. You have a bucket of bad units? Who cares? Grab the other bucket. But in the long-run you haven't done anything about the problems, and the fact that you have so much inventory there means you may not even know those problems exist, so the Japanese say, 'Get rid of your inventory. It's going to force you to run into these rocks, recognize your problems, which is a good thing not a bad thing because once you know the problems are there, you can fix them. When you fix them, there are no rocks. Guess what? When there are no rocks, you don't need any inventory, so you become really competitive."

Okay, so there are pros and cons and the question is then how much to order. As I say, the very basic model is called the economic order quantity model, and we're trying to figure out how much to order every time that the item is ordered. There are a number of assumptions involved with this model, and when you hear them all, you may say, "This is crazy. No product satisfies all these assumptions." But the good thing is that the EOQ is a pretty robust answer, so it's

usually a pretty good answer even if some of the assumptions are violated, but basically what happens is if an assumption is violated, you go find the model that handles that case when that assumption isn't true and use the more complicated one, but this is the one that gets us started, and this is the most basic inventory model we have.

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Basic Economic Order Quantity (EOQ Model)

Problem: How much of a given item to order every time that the item is ordered.

Assumptions

1. The time horizon is infinite.
2. Demand rate is constant over time (e.g., 10,000 units/year).
3. Demand rate is uniform (i.e., demand occurs continuously and smoothly).
4. Purchasing lead time – 0, and the order arrives in one lot (infinite production rate).
5. No constraints on the order size.
6. Decisions for one item are made independently of decision for other items.

[Chuck Munson]: So here they are: number one, the time horizon is infinite. So we're assuming that demand is 1,000 per year, year after year, until the sun blows up. Number two, the demand rate is constant over time, so it never changes. It's going to be 10,000 units every year or year after year until the sun blows up. Three, the demand rate is uniform, which means it occurs continuously and smoothly, so that implies no lumpiness in demand and no seasonality of demand. Number four, purchasing lead time is zero, so as soon as we order it, it arrives like Star Trek, and the order arrives in one lot, so we can produce it infinitely quickly. That may be true for ordering. If you're making it, it may take you time to make it. We have a model for that in chapter 12; it's called the production order quantity model. I encourage you to check it out to see what the difference is. Number five, no constraints on the order size, so theoretically you could order ten years' worth of material now. Number six, decisions for one item are made independently of decisions for other items. That may be problematic if you have a budget constraint for purchasing or if you have a warehouse base constraint and you're running out of space in your warehouse for all your items.

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7. Deterministic world (i.e., no uncertainty in demand, lead time, or supply).
8. No backorders are allowed.
9. The order quantity can be a fraction.
10. Purchased price does not depend on lot size.
11. Cost factors remain constant over time.

[Picture of a graph]

[Chuck Munson]: Number seven, no uncertainty. That means deterministic world, no defects, demand will be what we think it will be, perfect forecast, etc. Number eight, no backorder, so we have to fill demand as it comes. Number nine, it's okay to have a fraction as the order quantity, and what we typically do is round this to the nearest whole unit, and that's okay. It's a very

robust answer in that sense. Number ten, the purchase price does not depend on lot size. What that means is for the EOQ we assume no quantity discounts. Chapter 12 does show how you handle the quantity discount situation. And finally, cost factors remain constant over time, which is no inflation. So as I say, you may think that's very restrictive and it is, but to the extent that those assumptions aren't violated too badly the EOQ is a robust answer, a robust model, that does quite well.

Here's a little picture of what's going on: because of these assumptions we order Q and then because demand goes down smoothly and continuously over time it's a straight line until we hit zero, takes no time to get here and it all comes at once, so the line goes right back up to the top- down, up, down, up, so you get this solid tooth pattern over time. Well if that's what's happening, then your average inventory is Q/2, okay? Look what happens below, what's going on? The Q is higher, which means we're ordering more, but that also means it takes longer to deplete, and so that's the tradeoff in order size. The more you order, the more you have on hand. Average inventory is higher here than it is up here, but you're ordering less frequently, so you don't pay as many setup costs, okay?

Next we'll talk about the cost elements of the EOQ model. For the basic EOQ we ignore purchasing cost of the units. Not that we don't pay it but if purchasing cost doesn't depend on the lot size there is no reason to include it in the model, right?

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Cost Elements of the EOQ Model

Purchase cost is ignored since it does not vary with lot size and demand is known.

We consider 2 costs:

1. Setup costs per order ,and
2. Inventory holding costs.

<u>Setup Costs</u>

- Time needed to prepare a purchase order
- Receiving and inspection
- Order forms
- Postage
- Telephone calls
- Authorization
- Vendor's fixed charge

<u>Inventory Holding Costs</u>

- Warehouse space
- Interest on tied-up money
- Obsolescence, breakage, spoilage, deterioration
- Taxes
- Insurance

- Pilferage
- Worker's compensation costs

[Chuck Munson]: So the two costs that do matter are the setup costs per order and inventory holding costs. Setup costs are also called ordering costs, and this is what you have to pay every time you place an order or set up a batch for production, and it could include all these things: the postage, the order forms. A big one can be a fixed charge by the supplier so this could be a fixed shipping and handling charge, it could be 10,000 dollars for your supplier to set up, to run the batch for you, etc. So this is independent of the size of the order, okay?

Inventory holding costs are also called carrying costs, and the two biggest probably are the first two. One is the space that you're paying to store things, and even if you own a warehouse, if you can cut your inventory in half maybe you could lease that space to other people, other companies or find other uses for that space, or if you're renting space it's clear what that's costing you.

Another important one, probably the most important one is this one, and that's the interest on the tied-up money. If you've had an economics course, you might have heard of opportunity cost of capital, or if you've had a finance course we often use the weighted average cost of capital for this, and the idea is something like the following: suppose I own a store that sells Disney items, and I pay my supplier 100 dollars for a little Mickey Mouse porcelain thing and I stick it on my shelf, so part of the issue is I don't have that 100 dollars to spend on other things, but it's not just the 100 dollars, it's the interest I could have gotten on the 100 dollars. So if Mickey Mouse stays in my store for a year, a year later I still have a 100 dollar Mickey Mouse and my counting books show that, but what happened in the meantime was I could have taken the 100 dollars, reinvested in my company, and earned my return on that. At a minimum, I could have put the 100 dollars in the bank and could have- of course we're not earning much these days, but suppose I earn 5% on my investments; at the end of the year I would have had 105 dollars. So if my return is 5%, that's how much it costs on everything that I hold in inventory, so if I double the amount of money I have inventory, I double the amount of interest that I'm losing, so it's an opportunity cost of using that money to put something in storage instead of using that money to make money, okay?

Other items in terms of holding costs: breakage, spoilage, deterioration, taxes, insurance- if you think about automobiles, the auto dealers don't pay what you and I pay, but they still have to insure every car on their lot. Pilferage, this could be customers stealing your goods or even sadly, employees stealing goods and workers compensation costs as well. Believe it or not, the biggest workers compensation cost is back injuries due to moving inventory, so the more inventory you have to move the more likely it is to have workers compensation costs.

Okay, so if we put these two costs together, they go in different directions. The holding cost goes up linearly in quantity, so if I have twice as many units, I'm going to pay twice as much holding cost. On the other hand, the more I order at a time the less often I order, and so my annual setup costs go down. The total cost is the sum of those two, and when you plot them on a graph it's kind of a U-shaped convex function, and we're interested in finding the minimum of that. Minimum happens to be the very point where those two cost functions cross, and that is what we call the EOQ.

Okay, so that's a lot of background. Let's just go quickly through the formulas and a couple of examples, and then we'll wrap this up.

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EOQ with *Assumption 1: Holding cost is expressed <u>per unit</u>*

Let:

D= annual demand

S= setup cost per order

H= holding cost per unit per year

Q= number of units per order (decision variable)

Q* = optimal order quantity

Number of orders per year:

Average inventory:

Total Cost= annual setup cost + annual holding cost

[Chuck Munson]: The basic model assumes that holding cost is expressed per unit per year, so for example this motor costs me 10 dollars per unit per year to hold in inventory. The D will be annual demand. That has to be set up cost per order and Q is the decision we're trying to make, that's the number of units per order. When we see a little star that means the optimal or best one.

How many orders per year do we have? Well if annual demand is D and we're ordering Q, we will order D over Q number of times. If demand is 12,000 a year, we order 1,000 at a time, we'd order 12 times per year or about once per month.

What's the average inventory? Well as our graph showed on the previous page it is Q over 2, so once we know those two things we can write out our total cost function. The annual setup cost would be what? D over Q, which is the number of orders per year times how much it costs to place an order, right? So that's the annual setup cost, and your holding cost is how much we have on inventory on average over the year times H, the cost to hold one unit in inventory for one year. So this as a function of Q is that U-shaped function, and it turns out you can find using calculus or other means that the optimal order quantity is the square root of 2 times D times S over H. That's the magic EOQ.

And what I don't think the book shows you but I'll show you here- if you plug this into Q, so if you know you're ordering the EOQ and do some algebra, it turns out that the total cost at that point is the square root of 2 times D times S times H, so notice it looks very much like the EOQ except the H is in the numerator. It also tells us what happens to total cost as things change. If my setup costs per order doubles, my total cost will go up by the square root of two, etc. Okay? So this TC is the total cost for any Q. This TC is the total cost for the EOQ, which is this, okay?

There's another way to express these. Sometimes instead of holding costs being given as H, it's been given as an I, an annual holding cost percentage of the price or the cost per unit to you. So in this case H simply equals I times P. This would particularly be a good model to use if the main part of your holding cost is that opportunity cost of capital because then it's really that

percentage times the cost to you of the unit and that I then is applied to all the inventory that you buy even if they cost very different amounts, so everything's the same as the other previous page we just replace H with IP, so the total cost is D over QS plus Q over 2IP, and the EOQ is square root of 2DS over IP, right? And the total cost at Q=Q* is the square root of 2DSIP.

Alright with those formulas in hand, let's do a couple of examples, and then we'll be done with EOQ.

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EOQ Theorem

At the optimal order quantity, the ordering cost equals the holding cost. ‘

Example 1: D= 12,000 units/year

S= $60/order

H= $10/unit/year

Holding Cost= Q*/2 (H)=

Setup Cost= D/Q* (S)=

Example 2: D= 48,000 units/year

S= $20/order

I= 18%

P= $100

[Chuck Munson]: As you saw on the graph earlier, this is true mathematically that the optimal order quantity, the ordering cost per year, equals the holding cost per year, so we're really finding that balance, and that's a good way to double check your answer. Your annual holding cost should equal your annual setup cost if you're ordering the EOQ. If it doesn't, you have made a mistake somewhere, so let's look at the example here: annual demand is 12,000 units, setup cost is 60 dollars per order, and holding costs is 10 dollars per unit per year. So the EOQ is that formula that we just went over: 2 times 12,000 demand times the setup cost of 60. You divide that by the H, which is 10. That reduces to the square root of 144,000, 379.47. So in practice you would typically round this to 379 and rounding is fine. If you're talking about an airplane and your answer's 2.4 then there's actually a formula that tells you whether you should round that to two or three, and an airplane is so expensive that it does matter, but for most everything else just round to the nearest whole unit.

Okay, let's look at our costs, see what happens, and I'll go and keep the two decimal places just to reduce the rounding errors a little bit. The holding costs would be Q over 2 times H or $1897.35. The setup cost would be D over Q, 12,000 over 379.47 times that 60 dollar setup cost. That's $1897.38. So the only reason I didn't- they're not exactly equal to the penny is that I

rounded the Q itself to the second decimal place. But these are the same for all intents and purposes, so that is good news for us. That means we probably didn't make a mistake. If you add those two together you get $3794.73, so that's how much it costs us each year for holding and setup costs ordering 379 units. If we order any more or any less, this will go up. So this is the cheapest way to do it given our cost structure.

But for completeness we can do the shortcut formula for total costs. That's the square root of 2 times 12,000 times 60 times 10, and you'll find out that if you do that, you'll get the exact same thing, $3794.73.

Alright, a quick example with the I: so in this case we weren't given an H but we were told that holding cost is 18% of the price of the product, and by the way the I is always times what it costs you not what you're going to sell it for because the opportunity cost is based on what you paid, not what you're going to eventually be paid. So I have a demand and a setup cost, so same idea, Q* is the square root of 2 times 48,000 demand times a setup cost of 20 dollars each time over 18% of 100 dollars, so H is 18 dollars in this case. That will reduce to 326.6 units, and if you want to check on your own, the total cost of that is $5,878.78, alright?

So that's a quick review of inventory and EOQ. Thank you.